DOCUMENT RESUME

ED 389 549 SE 057 192

AUTHOR	Gallardo, Aurora
TITLE	Negative Numbers in the Teaching of Arithmetic. Repercussions in Elementary Algebra.
PUB DATE	Oct 95
NOTE	8p.; Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (17th, Columbus, OH, October 21-24, 1995). For entire conference proceedings, see SE 057 177.
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Algebra; *Arithmetic; Foreign Countries; Mathematics Education; Questionnaires; Secondary Education; *Secondary School Mathematics; *Subtraction; *Whole Numbers

ABSTRACT

This article reports the results of a questionnaire applied to secondary school students (n=35) to explore efficiency in the resolution of equations in the domain of whole numbers and the spontaneous responses to problems leading to negative solutions. The most significant results obtained with the questionnaire are the lack of knowledge of the double use of brackets in arithmetic expressions, the partial comprehension of the operation of subtraction, and the difficulty in the operativity of expressions with double signs. The conclusions of this research suggest recommendations for the teaching of whole numbers. Contains 13 references. (Author/MKR)

Negative Numbers in the Teaching of Arithmetic. Repercussions in Elementary Algebra

Aurora Gallardo

A Paper Presented at the Seventeenth Annual Meeting for the Psychology of Mathematics Education (North American Chapter)

October 21-24, 1995

NEGATIVE NUMBERS IN THE TEACHING OF ARITHMETIC. REPERCUSSIONS IN ELEMENTARY ALGEBRA.

Aurora Gallardo, Centro de Investigación y de Estudios Avanzados del IPN, Mexico

This article reports the results of a questionnaire applied to 35 secondary school students in order to explore the efficiency in the resolution of equations in the domain of whole numbers and the spontaneous responses to problems leading to negative solutions. The most significant results obtained with the questionnaire are the lack of knowledge of the double use of brackets in arithmetic expressions, the partial comprehension of the operation of subtraction and the difficulty in the operativity of expression with double signs. The conclusions of this research suggest reccommendations for the teaching of whole numbers.

The Study

The present work describes the first stage of the project *"The Status of Negative Numbers in the Resolution of Equations"* (Gallardo, 1994a). This project deals with the study of negative numbers in their interaction with the languages and methods used to solve equations and problems. Other stages of the project are described in Gallardo & Rojano (1993, 1994) and Gallardo (1994b). This article reports the results of a questionnaire responded to by 35 secondary school students in order to explore proficiency in the resolution of equations in the domain of whole numbers and the spontaneous responses to problems leading to negative solutions. The interest in reporting the first stage of the project is the importance that the results obtained by the use of this questionnaire have for the teaching of whole numbers in the field of arithmetic and their later repercussions in elementary algebra. The questionnaire was responded to by students aged between 12-13 years, before they had received any formal algebra teaching, and covered the following topics:

1. *Operativity in the domain of whole numbers at the syntactic level and their representation in the number line.* The student is asked to solve additions and subtractions with whole numbers using the number line. The most difficult exercises were the following[1] : $a - (-b) =$ and $- a - (-b) =$ with a, b natural numbers. The students obtained a percentage of correct answers ranging from 37% to 6 % on the above items.

Regarding the operativity of whole numbers at the syntactic level, exercises of the following form were designed: $a + b =$, $a \bullet b =$, $-(a + b) =$, $(-a) + (-b) =$, and $a - (b-c-d) =$, with a, b whole numbers. Furthermore, the student is asked if the following expressions are true or false: $a + b = a + b$; $(a)(-b)=(-b)(a)$; $-a (b (c)=- (a (c) (b$; $a - (b-c)=(a-b)-c$; $a (b-c)=ab - ac$ with a, b natural numbers. Marks below 30%

[1] All the examples in the questionnaire are numerical.

were obtained with this type of exercises. The greatest difficulty was due to the erroneous operativity of the minus sign together with the inadequate use of brackets.

2. *Location of the symmetric of a number in the model of the number line.* On the items corresponding to the symmetrics of numbers: $-(+a)$, $-(-(+a))$ with a as natural number, marks of 40% were obtained. The most difficult exercise of this theme corresponded to the symmetric of $(-(-a))$, that is, with a double minus sign. The percentage of correct answers in this case was 20%.

3. *Order in whole numbers.* The following questions were to be answered on this theme:

   Order these numbers from smallest to largest: -4, 3, 0, 14, -3, -8.

   Write three whole numbers greater than -3.

   Write three whole numbers smaller than -7.

   Write a whole number between -3 and -7.

   Write a whole number between -1 and 2.

   How many whole numbers are there between -5 and 0?

   How many whole numbers are there between -4 and 8?

The exercises 4 and 5 obtained the highest number of correct answers (from 83% to 77%). Item 6 achieved a higher percentage (66%) than item 7 (57%). The latter exercise is more difficult because *"you have to pass through zero"*. Exercise 1 corresponds to the order which *"appears natural to the student"*, that is, to order from smaller to larger (43% correct answers).

4. *Translation into symbolic language of situations expressed in words.* In these exercises different situations were presented and the student was asked to describe them using whole numbers. The following illustrates some of these situations:

   The temperature is 20 degrees below zero.

   Jose won 2 500 pesos.

   Archimedes was born in the year 267 before our era.

   The school is owed 25 000 pesos.

   Rosa neither won nor lost.

The lowest percentage of correct answers corresponded to item 5 (68%).

5. *Use of pre-algebraic languages in the context of equations.* In these excercises the student is asked to solve equations with the form: $\square$

$\pm a = b$, $a \bullet \square = b$ and $a \bullet \square \pm b = c$, with *a, b, c* whole numbers. The most difficult items (40% correct answers of the total) were those where the number sought is negative.

6. *Resolution of word problems*. These problems revealed that the student has difficulty in formulating a subtraction when the statement contains the word difference. In the same way, problems with negative solutions are complicated for the students. In the latter case the percentage of correct answers is 7% (see results 3 and 4 of this article).

Results of the Study

Among the most significant results obtained with the questionnaire are the following:

1. *Lack of knowledge of the double use of brackets in arithmetic expressions.* Students are unaware that the bracket can be used as a symbol for grouping terms in an additive situation and as a multiplicative operator. To illustrate this we can take as an example one of the items on the questionnaire where students are asked to decide if the equality $20-(7-8)=(20-7)-8$ is true or false. Observe that in the first side of this equality the bracket indicates the grouping of 7-8. Moreover, the same bracket expresses a multiplication by -1, denoted by the minus sign which precedes it: -(7-8). The operativity is carried out with whole numbers. However, in the second side of the previous equality, the operations are effected in the domain of natural numbers and the brackets indicate only the grouping of 20-7.

These facts, which are not taken account of in the teaching of arithmetic, are inherited by algebra. The student does not understand expressions such as the following:

$$(x-y) + (w-z) = (x+w) - (y+z) \qquad (1)$$
$$(x-1)^2 = (x^2-2x) + 1 \qquad (2)$$

In the first side of (1) the brackets group terms and in the second side the bracket is used as a multiplicative operator (observe the minus sign in front of the second bracket). Again, in the first side of (2), the brackets indicate squaring an expression. In the second side, the brackets group the first two terms of a trinomial. In the terrain of algebra the situation becomes more complex because the literals do not reveal the numerical domain to which they pertain. It is very important to warn the student from the outset, that is, from the teaching of arithmetic, with which numbers they are working in the exercises they do.

2. *Wrong resolution of pre-algebraic expressions.* There is greater difficulty in equations of the form $\square \pm a = b$, $a \bullet \square = b$ and $a \bullet \square \pm b = c$ with *a, b* whole numbers when the value sought is negative. This situation permits the conjecture that a place-holder will contrib-

ute to the avoidance of the negative solution in algebraic equations. The place-holder has the inherent connotation of *"being filled"*. The student generally seeks to *"fill it"* with a positive number.

3. *Partial comprehension of the operation of subtraction.* The student does not solve word problems which indicate the difference between two whole numbers with a subtraction. Moreover, he/she erroneously conceives situations of *"complete to"* as in the following item: *"A person is going to copy part of a book, from page 29 to page 35. How many pages does he copy?"*. The majority of students effect the subtraction 35-29, that is, they understand the subtraction as "take away".

4. *Abandoning of arithmetic methods and use of literals in the formulation of word problems when the solution is negative.* This situation is found in those problems with an evident contradiction in the statement if the student supposes that the solution is positive. An example of this is the following problem: A says to B: *"If you give me all your money and I add it to mine, I can buy a horse which costs 1000 units"*. B answers A: *"If you had three times what you have and I had double what I have, altogether it would add up to the price of the horse"*. How much money did each friend have?

The student writes the response *"it can't be done"*, or else *"who do I pay attention to, A or B?"*. Other students try to formulate an equation even though they have not had any formal algebra teaching.

5. *Arithmetic methods make one of the conditions of some word problems unnecessary.* This happens, for example, in the following problem: *"A salesman has bought 15 pieces of cloth of two types and paid 160 coins. If one of the types cost 11 coins the piece and the other costs 13 coins the piece, how many pieces did he buy of each price?"* Fifteen students look for multiples of 11 and 13 that add up to 160 (this is equivalent to solving the equation $11x + 13y = 160$. The existence of $x + y = 15$ is ignored). The students do not see that one of the two solutions is negative. However, this problem can be solved arithmetically, changing the data in the statement in order to obtain contradictory facts and provoke a conflict. In this research, conflict is achieved by decreasing the numerical data 160, 15, 13 and 11 to 40, 3, 3, and 2 (see Gallardo & Rojano, 1993).

The considerations expressed above show the necessity of solving, via teaching, the difficulties presented with negative numbers in the field of arithmetic, before students begin formal algebra courses.

In the research literature, authors such as Glaeser (1981), Bell (1982), Freudenthal (1983), Fischbein (1987), Janvier (1985) and Peled (1989), among others, have indicated the conceptual and operative problems which arise during the process of teaching-learning of negative numbers. Specifically, Vergnaud (1989)

points to the obstacles that these numbers represent when they are introduced into teaching, preceding the study of algebraic concepts. On the other hand, expressions such as (1) and (2) above are analyzed as propositional rules lacked visual salience (Awtry & Kirshner, 1994).

Recommendations

The conclusions of the research described in this article suggest the following recommendations:

1. To inform students of the numerical domain of the expressions to be dealt with, explaining the double use of brackets.
2. To use the method of teaching by diagnosis, which implies the identification of errors and false conceptions in a topic and later the formulation of a design for teaching in which the difficulties are exposed and solved through discussion-conflict.
3. To encourage the use of teaching models with whole numbers, different from the model of the number line, which permit other interpretations of the negative number, different from that of positions or displacements. We suggest the use of discrete models where the whole numbers represent objects of an cpposing nature (protons, electrons; black balls, white balls, etc). In these latter models, the sign of operation is distinguished from the sign of number in the case of double signs *[-(-a), +(-a)* and *-(+a)]*.
4. In the resolution of pre-algebraic equations, we recommend the teacher not restrict his/her use of the place-holder as *"a place to be filled by a number"* but to encourage the methods of inversion of operations which propitiate *"operating the unknown"* and the extension of the numerical domain of solution in the terrain of true algebra (Filloy & Rojano, 1984).

References

Awtry, T. & Kirshner, D. (1994). *Visual salience in algebraic transformations.* Proceedings of the XVI Annual Conference for the Psychology of Mathematics Education, North American Chapter, Louisiana State University, USA, Vol. 1, pp. 131-137.

Bell, A. (1982) *Looking at children at directed numbers.* Mathematics Teaching. No. 100.

Fischbein, E. (1987). *Intuition in Science and Mathematics. An Educational Approach.* Reidel, Holland, Chapter XII: The practicality of intuitive meanings, analysis of an examen: The negative number. pp. 97-102.

Filloy, E. & Rojano, T. (1984). *From arithmetical to an algebraic thought: A clinical study with 12-13 year olds.* J. Moser (Editor). Proceedings of the VI Annual Conference for the Psychology of Mathematics Education, North American Chapter. pp. 51-56, Madison, Wisonsin, USA.

Freudenthal, H. (1983). *Mathematics as an educational task.* Chapter XII, Reidel Droodrecht, The Netherlands.

Gallardo, A. (1994a). Doctoral Dissertation. *El estatus de los números negativos en la resolución de ecuaciones algebraicas.* Centro de Investigación y de Estudios Avanzados del IPN, Departamento de Matemática Educativa, México, 2 Vols.

Gallardo, A. (1994b) *Negative numbers in algebra: The use of a teaching model.* João Pedro Da Ponte & João Filipe Matos (Editors). Proceedings of the XIX Annual Conference for the Psychology of Mathematics Education, Vol II, pp. 376-383, Lisbon, Portugal.

Gallardo, A. & Rojano, T. (1993). *Negative solutions in the context of algebraic word problems.* Proceedings of the XV International Group for the Psychology of Mathematics Education, University of Tsukuba, Japan; and North American Chapter, San José State University, USA, Vol. 1, pp. 121-127.

Gallardo, A. & Rojano, T. (1994). *School algebra. Syntactic difficulties in the operativity with negative numbers.* Proceedings of the XVI International Group for the Psychology of Mathematics Education, North American Chapter. Louisiana State Univesity, USA, Vol. I, pp. 159-165.

Glaeser, G. (1981). *Epistemologie des nombres relatifs.* Recherches en Didàctique des Mathématiques, Vol. 2, No. 3, pp. 303-346.

Janvier, C. (1985). *Comparison of models aimed at teaching signed integers.* Proceedings of the Nineth Annual Conference for the Psychology of Mathematics Education, State University of Utrecht, The Netherlands, pp. 135-140.

Peled, I., Mukhopadhyay, S. & Resnick, L. (1989). Formal and informal sources of mental models for negative numbres. Thirtheenth Annual Conference for the Psychology of Mathematics Education. Paris, France. pp. 106-110.

Vergnaud, G. (1989). *L'obstacle des nombres négatifs et l'introduction à l'algèbre. Construction des savoirs.* Colloque International Obstacle Epistémologique et Conflict Socio-Cognitif, CIRADE, Montreal.